Some important photon experiments for schools in the light of quantum electrodynamics

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Abstract

Quantum physics classes in schools usually focus on some experiments with laser light, usually involving very many photons. But what happens if the same experiment is performed with only one photon at a time? Using the tools of theory, it is shown for the double-slit experiment, for polarizers - important for demonstrating basic facts of quantum physics [Hü] - and the cavity resonator that, apart from significant statistical variations, the same results occur. For an impressive experiment with a modified double-slit with polarizers, which is often extended in school to a "quantum eraser" with "delayed choice", it is investigated to what extent the school discussion is admissible. Answers to such questions are presented in the sense of quantum electrodynamics, however for "free" electromagnetic fields not coupled to charges. This paper summarizes results of a longer investigation.

1 The problem

Whether a photon is a particle or a wave is often discussed in physics didactics and in high school physics classes. This is not the subject of this article. Rather, it is intended to present what the accepted higher-level theory of quantum electrodynamics (QED) or - more comprehensively - quantum field theory (QFT) teaches about the photon and its "behavior".

2 What is a photon in QED?

For the interaction-free electric field, i.e., not coupled to electric charges, it is easy to say: in QED [Ma], a photon is a particular (abstract) excitation state of the quantized electromagnetic field. It is the lowest excitation state with a determinate (excitation) energy E, a determinate[1] momentum **p**, the determinate particle number N = 1, and an integer spin, all simultaneously measurable properties that the system has simultaneously. In the simplest case, the energy is $E = \hbar \cdot \omega$ and the momentum **p** is $\hbar \cdot \mathbf{k}$, where $\omega = 2\pi \cdot f$ is the angular frequency, **k** the wavenumber vector with magnitude $k = 2\pi/\lambda$, $E = p \cdot c$ and $\omega = k \cdot c$, respectively. The relation between energy and momentum $E = p \cdot c$ can be valid in relativistic physics only for massless particles. In $\hbar = h/2\pi$, h is Planck's quantum of action.

This includes conservation laws for energy, momentum, angular momentum, and particle number, but the latter only for interaction-free photons. With these properties, especially because the associated conservation laws hold, the excited state appears to behave in some ways like a classical particle. Localizability is not one of the defining properties of a photon. That is all that can be said about the definition of a photon. In particular, one cannot say anything about a location (position, "place") of the excited state. An excitation state has no location ([Pas] and literature sources there). Place is a meaningless term for it - thus also for a photon. And it has no sense to ask for the path of a photon, e.g. through a double-slit or a Mach-Zehnder interferometer: A photon belongs to the whole electro-

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[1] *In this article I use the word "determinate", where other authors prefer "certain", instead of the German word "bestimmt". The uncertainty principal prevents two conjugate observables to have determinate values simultaneously. Analogously is written: "indeterminate" (uncertain) instead of "unbestimmt".*

magnetic field. Likewise, the size of a photon is a meaningless concept for the excited state. Moreover, there are pairs of properties, which a photon cannot have simultaneously - unlike a classical system, e.g. two different components of the electric and magnetic field, e.g. E_x and B_y, or x and y component of angular momentum, anyway not location and momentum. Such interaction-free photons do not exist in nature, but they are a good starting point for further investigations by renormalization techniques.

The experimenter is more interested in where he can detect a photon frequently. For this purpose, a location-dependent photodetection probability amplitude [Scu] can be constructed, which depends on the light itself and on the charges in the detector. It is particularly large, for example, in the maxima of a double-slit experiment. It resembles the location-dependent probability amplitude for an electron. The occurrence of "granular" localized phenomena is only noticed experimentally when fields interacting with charges play a role, including in measurements. This is justified more exactly by the decoherence theory (e.g. [Zeh]), which is mentioned here only in the margin.

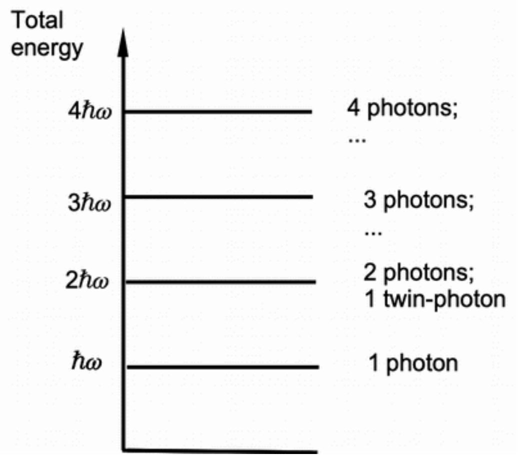

Fig. 1: *Excitation states of the electromagnetic field to uniform wavelength ǵ. Shown are the energy levels for the excitation of 1, 2, 3, ... photons. The lowest excitation state represents a photon with energy □·ï = □c/ǵ.*

3 How do we get there?

The way, roughly described in keywords, is the following: Maxwell's equations, vector potential **A**, expansion of the fields **E**, **B** and **A** into plane waves; quantization of the fields by introducing permutation relations which take into account that not all "classically conceivable properties" [Kü] of a quantum object are simultaneously determinate; operators, as it were "containers" for all information about the system; states to "extract" this information in a certain situation; formulation of Hamiltonian/energy operator H, momentum operator **P**, particle number operator N and spin **S** with creation and annihilation operators; photon as energetically lowest excitation state of the electromagnetic field at a certain frequency resp. wavelength; common eigenstate at the same time of H, **P**, N and spin **S**. There is no localization operator and therefore no probability to measure a photon near a certain location.

4 Fields are replaced by field operators

Maxwell's equations and their conclusions are kept in QED, but the fields are now abstract field operators, as well as quantities derived from them like the energy density $U = ½·\varepsilon_0\, [E^2 + c^2B^2]$. Only expectation values concerning certain states are related to measurable values and show properties of these states. The origin from Maxwell's equations explains in which way photons have "inherited" some "wave-like properties".

5 Excitation states of the electromagnetic field

The energy level scheme of the electromagnetic field in a mode of a certain wavelength consists of a "ladder" of equidistant values (fig. 1) with the lowest excitation state of energy $\hbar·\omega = \hbar·k·c$. Higher rungs of the "ladder" have energies $n·\hbar\omega$. They belong to excitation states with n = 1, 2, 3, ...

equal photons. But among the excitation states there are other particle states with a determinate particle number n, e.g. twin-photons of two "entangled photons". In contrast to particle states, coherent states are characterized by indeterminate photon numbers. Thus, if one measures the particle number in such a state, one obtains varying values with an expectation value <n>, i.e., for <n> = 1, e.g., n = 1, 0, 3, 0, 10, 1, 2, ... These states were discovered by Glauber [Gl] in 1963 and are sometimes called Glauber states, named after him.

In "entangled states" two or more particles of the same or different kind form a common total state. Only for the total state a few properties are fixed, e.g. the total momentum or the total spin. The contributing particles have no individual properties without a measurement. With random values these arise only as a result of a measurement, but in such a way that conservation laws according to the "birth certificate"[2] are kept. Audretsch [Aud] speaks of an entangled state "almost without properties". There is also the possibility that different subsystems of a particle are entangled into an overall state. For example, the total angular momentum of the shell and nucleus of an atom could define an entangled state, or orbital and spin angular momentum, or center-of-mass and relative systems. For twin-photons, only for the total state some properties are determinate, e.g. the total momentum. The properties of the individual photons resulting from a measurement arise randomly taking into account conservation laws for the properties of the total state. If after decomposition of an entangled state with total momentum 0 a photon with momentum h·**k** is created during a measurement, instantaneously the second photon must have momentum -h·**k**.

For all states, some random results with associated probabilities occur in measurements, but no location-dependent probabilities occur for interaction-free photons. For a single photon state with indeterminate momentum or indeterminate polarization the theory teaches the probability for the occurrence of a determinate momentum or polarization. Similarly, one can prove that for an entangled two-photon state with total momentum 0, arising, for example, from two photons with quantum numbers **k**,s and -**k**,s' or -**k**,s and **k**,s' [3], that in a measurement only one of the two pair combinations can occur at a time, with probability ½ each. For all other wavenumber-polarization-combinations the probability results 0. I.e. if in a measurement the momentum h·**k** is found, one immediately knows - as already mentioned - that the second photon can have only the momentum - h·**k**. This corresponds to the experiment which is the basis of the EPR paradox [Ein]. If one has measured the momentum of one photon, both momentums are automatically determinate, without a "spooky action at a distance", simply so that the defining total momentum is preserved.

6 Excitation states in other physics domains

A rope can be excited to different classical natural oscillations. After quantization one finds - quite corresponding - excitation states with definite energies. They are called phonons and are treated as if they were particles. The wavelength determines their energy, the number of phonons the amplitude of oscillation.

Also in solid state physics, one knows various excitation states of a crystal: again phonons, which represent the quantized form of classical natural oscillations, polaritons, which represent entangled states of phonons and photons, and many others, e.g. the Cooper pairs of superconducting physics.

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[2] *By* ***"birth certificate of the entangled state"*** *I mean the totality of determinate properties of the entangled total state, which must be kept even if after a measurement the entangled state is decomposed into individual/single particle states. If e.g. the total momentum is 0 and also the total angular momentum is 0, the individual/single particle states must have opposite momentum and angular momentum after the measurement.*

[3] ***"Triad of entangled systems"****: Entangled systems often* ***arise*** *from non-entangled individual quantum particles, do not themselves* ***consist*** *of (individual) quantum particles, but* ***decay*** *back into individual quantum particles when measured.*

7 To what extent does a photon appear as a particle? Is school practice justified?

According to QED, a photon is the lowest-energy excited state of the electromagnetic field and also an eigenstate of the Hamiltonian/energy operator, the momentum operator, and the total particle number operator. Conservation laws are found for energy, momentum, and total particle number, just as they are for classical particles. This is the only justification for treating excited states of the electromagnetic field approximately as particles, especially in school. They **seem** to be particles. In this respect, photons behave like classical particles, but with one major difference: they have no location and no size.

8 Attempt to use the "normalized energy density" as a substitute for a location-dependent probability density

So there is no probability density, or probability of measuring a photon near a particular location. But it turns out that the expectation value of the energy density operator $U(\mathbf{x},t) = \frac{1}{2}\cdot[\varepsilon_0 E^2 + B^2/\mu_0] = \frac{1}{2} \cdot \varepsilon_0 [E^2 + c^2B^2]$ is location dependent. Relating it - for n photons of the same kind - to a single one of them (by dividing with the total energy $n\cdot\hbar\omega$), we obtain a location-dependent quantity which, integrated over the whole space, gives 1, like the probability density. I call it "normalized expectation value of energy density", or "normalized energy density" for short. The "normalized energy density" $<U(\mathbf{x},t)>/(n\cdot\hbar\omega)$ is taken as a measure of the probability density of finding the energy $n\cdot\hbar\omega$ in the vicinity of the location $\mathbf{x}$. Just as for an electron $|\psi(\mathbf{x})|^2\cdot\Delta V$ represents the probability of measuring the electron in an environment of size ΔV of a location $\mathbf{x}$, $<U(\mathbf{x})>/(\hbar\omega)\cdot\Delta V$ for a photon (or after "normalization" for n equal photons) represents a measure of the probability of finding the energy $n\cdot\hbar\omega$ in ΔV. However, without a coupling of the photons to the electric charges of a photoplate or a detector, you would not notice anything about the location-dependent "normalized energy density". But it is plausible that at places of large energy density also the interaction with charges will be particularly large, so that also the photodetection probability [Scu] is large at this place, and a photon is often detected in a detector. In the following concrete examples, the correct predictions of the QED are concluded from the "normalized energy density". Thereby its function for qualitative reasoning is confirmed analogously to the probability density e.g. for electrons.

9 Resonance states of the cavity resonator

A cavity resonator is e.g. a cube-shaped box with ideally conducting walls. If electromagnetic energy is fed into it at a suitable frequency, it can resonate. Standing electromagnetic waves of large amplitude are formed inside the box. Already classically, resonant frequencies are obtained if waves are fitted in such a way that an integer multiple of half the wavelength $\lambda/2$ corresponds exactly to a length ℓ. Depending on the energy supplied, the cavity resonator can oscillate at any amplitude according to classical notions. One can quantum theoretically expect similar results. What is examined is the expectation value of the energy density U. If one has prepared a suitable state with n equal photons of suitable energy, the electromagnetic field will also oscillate in a natural mode if the wavelength satisfies the classical condition ($\ell = \lambda/2 \cdot m$, $m = 1, 2, 3, \ldots$, i.e. $\hbar\cdot\omega = \hbar\cdot c\cdot\pi/\ell \cdot m$) [Haro].

But unlike in the classical consideration, quantum-physically the squared amplitude can take only discrete values which are proportional to the photon number n. If ħ·ω is the energy of the ground-state (1 photon), then the electromagnetic field with n equal photons has the energy n·ħω. If the electromagnetic field had been prepared into a coherent state, then the squared amplitude could again take any value (proportional to the expectation value <n> of the photon number). The spectacular thing about the quantum theory of cavity resonance is that standing waves are already possible for 1 photon in the cavity resonator. This is exploited in the thought experiment of Scully, Englert, and Walther [Scu1], which involves a double-slit experiment with highly excited Rydberg atoms. With the photon left behind by the atom in one of two cavity resonators in front of the slits of the double-slit, the location of passage of the atoms is measured, unfortunately destroying the double-slit interference, but without significant mechanical "disturbance" of the atoms passing through. In another experiment, the excitation period of a highly excited Rydberg atom in the resonator was shortened or lengthened within wide limits (up to 30 s instead of the typical 10^{-8} s), depending on whether one tuned the resonator's resonance frequency to the transition frequency or detuned against it. In the first case, the cavity resonator is particularly fond of accepting the photon from the atom; in the second case, it is particularly reluctant.

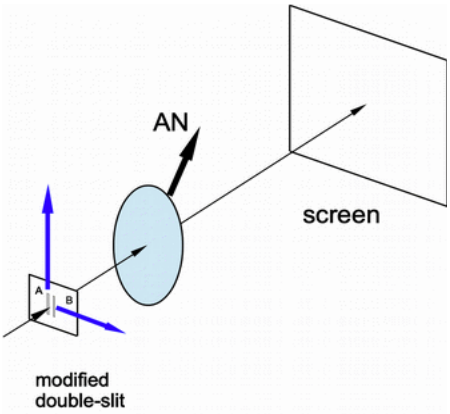

Fig. 2: *Modified double-slit: passing light is polarized differently depending on the slit. The analyzer AN is used to decide whether single-slit or double-slit interference is to be observed.*

10 Quantum theory of the double-slit experiment with one or more photons

The double-slit experiment with light works with a laser or largely monochromatic natural light. As a rule, very many photons are involved. What does the QED tell after calculation of the "normalized energy density" on the screen or the interference intensity? It is noticeable that at low intensity the interference figure builds up only gradually, i.e. on the screen single photons appear at randomly distributed places, but mainly at the places of maxima. Modern experiments show this more reliably than the historical Taylor experiment. The quantum theory distinguishes in which states one has prepared the used light. In particle states (Fock states) with a determinate[4] number of photons (i.e. n = 1, 2, 3, ..., 100000, 100001, ...) one notices the statistical fluctuations, but also that the expected value of the interference intensity near a certain location is proportional to the photon number n. This is spectacularly true even for a single photon - in Zeilinger's [Zei] parlance, in "single-particle interference" (as distinct from "wave interference"). If one performs identical experiments with one identical photon each, the interference figure of the double-slit gradually builds up from single events. In principle, one must also find expectation values for the interference intensity, which vary in steps proportional to n. If, on the other hand, one prepares the electromagnetic field into a coherent state (into a Glauber state) with indeterminate[5] photon number, then for any expectation value of the photon number, even, for example, at <n> = 0.5 , the field would behave like a classical field, except for the statistical fluctuations. This is even relevant for school physics when one wants to do photon counting experiments with laser light strongly attenuated by gray filters, e.g., in a demonstration of the Taylor experiment. However gray filters will never convert a coherent state into a single-particle state.

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[4] *In this article I use the word "determinate", where other authors prefer“certain”, instead of the German word “bestimmt”. The uncertainty principal prevents two conjugate observables to have determinate values simultaneously. Analogously is written “indeterminate” (uncertain) instead of “unbestimmt”.*

[5] *Measuring the photon number, in each equal time intervals, one would register varying photon numbers, once 0, another time 10, then perhaps 2, ... photons, though the expectation value of the photon number in the prepared state is e.g. <n> = 1.*

11 Quantum theory of the polarizer for one or more photons

If linearly polarized light falls onto a polarizer rotated by angle α, only the part $I_0 \cdot \sin^2(\alpha)$ of the incident intensity I_0 passes through the polarizer, depending on the angle of rotation α. This is stated by the classical Malus law, which is usually derived with a vector decomposition of the incident electric field intensity. On the other hand, one would like to demonstrate "basic facts of quantum physics"[6] in school, e.g. according to the "Würzburg Quantum Physics Concept" (WQPC) [Hü], with single photons by means of polarizers, although one has no single-photon source available. Fortunately, quantum theory shows that the Malus law is already valid for a single photon, as well as for particle states with n photons or for coherent states, i.e. with laser light. School experiments with laser light and polarizers therefore also show the behavior of a single photon.

12 Which-way information and interference in the modified double-slit? Is it a real quantum eraser? Is it a delayed choice experiment?

Perhaps you know the impressive experiment with a modified double-slit [Kü1], where one tries to "mark" the photons passing through slit A or slit B by means of differently oriented polarization foils (fig. 2, 3). It is claimed that the polarization of a photon measured later can be used to read off the point of passage. Then, however, the double-slit interference disappears, while the single-slit interference[7] still remains visible. Both seem to confirm the rule: "Which-way information and interference are mutually exclusive". Surprisingly, one can recover the double-slit interference if one wipes out ("erases" / "cancels") the information about the point of passage by letting the two "beams" pass together through a rotated analyzer ("quantum eraser"). Strongest contrast is obtained when the analyzer is set along the bisector between the orientations of A and B (fig. 3 below). If AN lets pass only one kind of photons, only single-slit interference is visible, otherwise the contrast can be changed continuously by the orientation angle. Sometimes one reads the incorrect assertion that by the analyzer setting one can force a continuous transition between the behavior of the photon "as wave" or "as particle". This is also sometimes erroneously seen as a subsequent, "delayed choice" for a wave-like or a particle-like behavior of the photons long after the "photons passed the double- slit". Some authors even discuss a so called “retrocausality”.

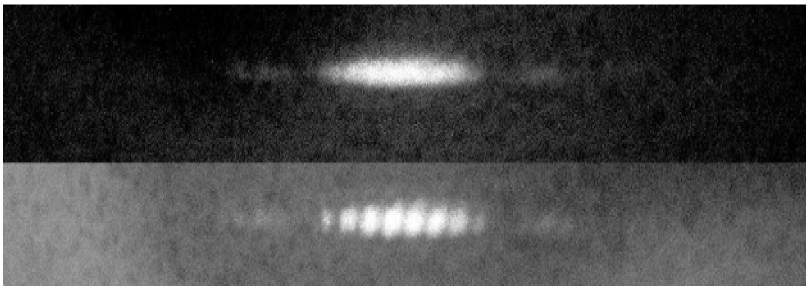

Fig. 3: Experimental result with a modified double-slit. Performed with laser light, this is not an actual quantum experiment. In the upper photo the analyzer AN is oriented in such a way that only the interference figure of a single-slit (SS) is visible, in the lower photo that of the double-slit (DS) with the envelope according to SS. But as shown in the more detailed text, it would also proceed in the same way with single photons. Glauber states, which in some way reconcile quantum theory and classical electrodynamics, teach that a semiclassical theory and experiment would show virtually the same result.

QED shows: In order to obtain the observed possibility of double-slit interference at all, one has to form the expectation value of the energy density operator U resulting from superposition of the two field operators with respect to a state with a *photon of indeterminate polarization* but with a determinate wavenumber vector.

After passing through the analyzer AN, a "normalized energy density" is obtained which, as expected, shows single-slit or double-slit interference with different minimum-maximum contrast depending on the orientation of AN. In particular, if the analyzer orientation corresponds to the polariza-

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[6] *For the **Würzburg Quantum Physics Concept** (WQPC), many online and print materials are provided at https://www.forphys.de/overview.html.*

[7] *The point of passage within the single-slit is always indeterminate.*

tion of slit A (or of slit B), one can only see single-slit interference (fig. 3, top). One is inclined to assume that then the photons must have come *through slit A* (or slit B). One believes to know now the path of the photon through slit A (or slit B) without seeing double-slit interference. This seems to fit the statement that which-way information and interference are mutually exclusive. But did the contributing photons really pass *through slit A* (or B)? The formalism shows: Through the double-slit *always* pass photons with indeterminate polarization and therefore indeterminate passage point independent of the orientation of AN. Only the analyzer AN turns them, depending on its orientation, into photons which sometimes correspond to the polarization of A, sometimes to that of B or to another one, in any case into photons which correspond to the orientation of AN. The photons get the respective polarization only when passing through AN! It is not a real which-way information. Nevertheless, the experimental result fits the mentioned rule, even if the true passage point is meaningless in this experiment, which cannot be decided. You now understand that one must not claim that one can decide, "long after passage", "between the wave nature or the particle nature" of the photon. It is not a real experiment of "delayed choice". In this respect also no real quantum eraser (quantum eraser) is shown. But an important aspect of the rule can be shown well, and if the teacher keeps these facts in mind, I think the experiment justifies the use in class. Also everybody can see that even the "interference figure of the single-slit" has nothing to do with a "photon as particle".

Similar considerations are valid for Wheeler's thought experiment [Whe] with the light of a quasar passing a gravitational lens in form of a galaxy, billions of light years away from us. Depending on the orientation or selection of a detector, it is supposedly decided today, close to us, whether photons billions of years ago are deflected "as particles" at each edge of an intervening galaxy, or together "as a wave". In reality the state of indeterminate place of passage remains maintained until the measurement next to us takes place. The measurement has no influence on the kind of the passage or the photon character at that time. That would be absurd.

13 Summary

In a longer version of this text [Hü1] is investigated what QED teaches about interaction-free photons. According to this, a photon is the lowest excitation state of the electromagnetic field with a determinate (excitation) energy, a determinate momentum, the determinate particle number 1 (and an integer angular momentum/spin), all simultaneously measurable properties that the system has at the same time.

In school a photon can be treated - without touching the question "wave or particle?" - as a particle, because it is an eigenstate of the Hamiltonian/energy operator, the momentum operator and the particle number operator. As for a classical particle, conservation laws for energy, momentum and particle number are valid, at least for interaction-free photons. Localizability is not one of the defining properties of a photon.

Further properties of such photons were examined in connection with some basic experiments of photon physics, which are also relevant for school physics. In a cavity resonator, in the double-slit experiment and in polarization experiments, classical results were largely reproduced, with the difference that their characteristics (e.g. energy density, intensity) show statistical properties and are proportional to the photon number, i.e. they depend on the photon number in discrete steps, showing varying results. Especially, however, it has been demonstrated that these experiments already work for a single photon, with results quite similar to the classical experiments, apart from unevitable statistics. A well-known school experiment with a modified double-slit, supposedly measuring the location of passage of the photon through the double-slit by its polarization, was analyzed. Although the experiment seems to confirm the rule "which-way information and interference are mutually exclusive", it is shown that it is not a true "which-way experiment", not even a true "quantum eraser". The path through slit A or slit B remains indeterminate, independent of analyzer orientation, until after exit from the analyzer.

It has been emphasized that for photons there is no localization operator and thus no probability of finding a photon in the vicinity of a particular location. Interaction-free photons do not have a location. Real photons have a location only when it is measured. This can be described by a location-dependent “photodetection probability amplitude” depending on the light itself and on the interaction with the detector. However this is beyond the scope of this paper.

To achieve some parallelism with the quantum mechanics of mass particles, it has been proposed here for qualitative discussions to replace the probability density of mass particles with a "normalized energy density" for photons, which is not unappealing because its expectation value has much in common with the classical quantity. Concrete examples that are also relevant for school lessons, seem to confirm this.

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